# A Theory of Boundary-Layer Flow Noise, with Application to Pressure Gradients and Polymer Solutions

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**A quantitative analysis, patterned after the qualitative work of Gardner and White, is made for wall pressure fluctuations caused by turbulent boundary-layer flow. The derivation uses the linear source contributions to pressure and results in a formula from which the statistical properties of wall pressure can be calculated by direct quadrature. The theory shows that there are four contributions to wall noise: 1) the wall friction, 2) the shape of the normal velocity spectra, 3) the magnitude of the normal velocity derivative ($\partial v/\partial x$), and 4) the local Reynolds number. Comparisons with Newtonian data for wall noise in zero and adverse pressure gradients show good agreement. Finally, based on sketchy velocity data for polyox and CMC solutions, a tentative calculation predicts that such additives do not cause wall noise reduction.**

## Nomenclature

$a, b, C$ = curve-fit constants in Eq. (16)
$B$ = law of the wall constant, Eq. (1)
$\Delta B$ = increase in $B$ due to a polymer additive, Eq. (3)
$C_f$ = skin-friction coefficient = $2\tau_w/\rho U_\infty^2$
$f(k, y)$ = Hankel transform of $R_v$, Eq. (12)
$f_u, f_v$ = longitudinal and normal velocity spectra
$H(y)$ = zero intercept of $R_v(h)$ — see Fig. 3
$P, p$ = mean and fluctuating pressure
$R_x$ = $U_x/\nu$ = local Reynolds number
$R_v, R_p$ = velocity and pressure correlation functions
$U(y)$ = longitudinal mean velocity
$u, v, w$ = fluctuating velocity components
$v^*$ = wall friction velocity = $(\tau_w/\rho)^{1/2}$
$v_0^*$ = polymer threshold friction velocity, Eq. (3)
$x, y, z$ = longitudinal, normal, and lateral coordinates
$y^*$ = $y/\delta$
$Z$ = dimensionless friction = $U/v^* = (2/C_f)^{1/2}$
$\alpha$ = Meyer's dimensionless polymer constant, Eq. (3)
$\delta$ = boundary-layer thickness
$\delta^*$ = displacement thickness
$\mu$ = fluid kinematic viscosity
$\rho$ = fluid density
$\tau_w$ = wall shear stress
$\xi, \eta, T$ = longitudinal, lateral, and time separations

***Superscript***

$(\ )^+$ = law of the wall variable

## Introduction

THE analysis of skin friction in the turbulent boundary layer is now well established for Newtonian flow. For arbitrary pressure gradients, one may use any of several finite-difference methods, e.g., Mellor,[1] or one may make rapid estimates of $C_f(x)$ with good accuracy with the integral method of White.[2] For the flat plate, there are several empirical expressions for $C_f$ in the text by Schlichting,[3] but an exact analysis is possible if one assumes that the familiar logarithmic law of the wall is valid for turbulent flat-plate flow:

$$u^+ = u/v^* = (1/k) \ln (yv^*/\nu) + B \tag{1}$$

where $v^* = (\tau_w/\rho)$ is the shear velocity. The constants are usually taken as $k \doteq 0.4$ and $B \doteq 5.5$. Using Eq. (1), Kestin and Persen[4] derived an exact formula for the local skin friction on a flat plate:

$$R_x = (k^{-3}e^{-kB})e^{kZ}(k^2Z^2 - 4kZ + 6) \tag{2}$$

where $Z = U/v^* = (2/C_f)^{1/2}$. Equation (2) is thus an implicit relation for $C_f$ as a function of local Reynolds number $R_x$ in Newtonian turbulent flow. The constant $(k^{-3}e^{-kB}) = 1.73125$.

In sharp contrast to these highly refined skin-friction analyses, the theory of turbulent wall pressure fluctuations is almost nonexistent. A comprehensive review of wall pressure measurements is given by Corcos,[5] and simplified estimates of mean wall pressure are given by Kraichnan[6] and Hodgson.[7] A later theory by Gardner,[8] extended and improved by White,[9] gave fair estimates of the shape of the normalized space/time pressure correlations but could not calculate absolute magnitudes. To date, no theory has been preoposed for estimating accurately the statistical properties of wall pressure under varying conditions such as pressure gradient or the presence of dissolved polymers. It is the purpose of the present paper to present such a general, albeit simplified theory of turbulent wall pressure.

One application of the present theory will be to estimate the effect of aqueous polymer solutions on wall pressure. The ability of polymer solutions to reduce skin friction, first discovered by Toms[10] in 1948, has now received intensive study. Comprehensive reviews of skin friction reduction (the "Toms effect") are given by Hershey[11] and by Shin.[12] An explanation for the friction reduction which is satisfying mathematically, if not rheologically, was given by Meyer,[13] whose velocity measurements showed that the constant $B$ in Eq. (1) changes systematically with polymer type, polymer concentration, and wall shear stress. Meyer showed that $B \doteq 5.5$, its Newtonian value, until the shear velocity $v^*$ reaches a threshold value $v_0^*$ which is approximately independent of concentration for any given polymer. Meyer further suggested an empirical correlation for $B$:

$$\begin{aligned} B &= 5.5 \qquad (v^* \leq v_0^*) \\ B &= 5.5 + \Delta B,\ \Delta B \doteq \alpha \ln (v^*/v_0^*) \quad \text{if} \quad v^* > v_0^* \end{aligned} \tag{3}$$

The parameter $\alpha$ is dimensionless and depends upon the weight concentration of the polymer. It does not depend upon the shear stress until $v^*$ becomes very large. The threshold shear $v_0^*$ is approximately constant for a given polymer. Figure 1 shows measured values of $\alpha$ and $v_0^*$ for polyox (WSR-301) and for guar gums as taken from Refs. 14–18.

Received April 26, 1968; also presented as Paper 68-642 at the AIAA 4th Propulsion Joint Specialist Conference, Cleveland, Ohio, June 10–14, 1968; revision received December 19, 1968. This work was supported by Code 2112 of the U.S. Navy Underwater Sound Laboratory and by Office of Naval Research Contract Nonr 396(14).

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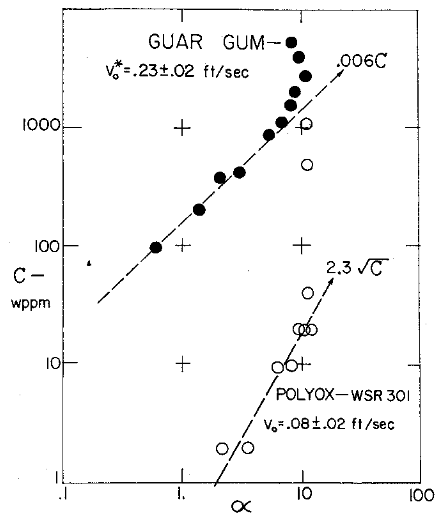


**Fig. 1 Measured values of $\alpha$ and $v_0^*$ for two types of polymer additives.**[14–18]

The maximum value of $\alpha$ is seen to be about 10. McNally[14] has pointed out that Meyer's correlation, Eq. (3), is only fair for polyox, and Fabula[19] has suggested that it is more realistic in the case of polyox simply to correlate $\Delta B$ with shear stress for various concentrations without attempting a logarithmic fit.

Based upon combining Eqs. (1) and (3) with the momentum equation, White[20] has derived the following formula for flat-plate flow with a polymer solution:

$$R_x \doteq 1.73225 e^{kZ}(v^*/v_0^*)^{k\alpha}(k^2Z^2 - 4kZ + 6) \qquad (4)$$

This formula will be needed in the flow noise analysis which follows. By comparing Eqs. (2) and (4), we see the local $C_f$ in polymer flow at a given $R_x$ is equal to the Newtonian $C_f$ evaluated at an effective Reynolds number $R_N$ given by

$$R_N = R_x(v^*/v_0^*)^{k\alpha} = R_x e^{k\Delta B} \qquad (5)$$

a relation which is also true for pipe flow if $R$ is the diameter Reynolds number. If, for plate flow, we assume that the Newtonian $C_f \propto R_x^{-1/7}$, Eq. (5) shows that the polymer reduces skin friction in the ratio $e^{-k\Delta B/7}$, which equals 0.24 if $\Delta B = 25$ (the maximum value attainable by present polymers). Thus, polymer solutions can reduce friction as much as 76%. As we shall see, friction reduction is one of the two chief mechanisms of noise reduction revealed by the present analysis, the second being changes in the spectrum in the velocity fluctuations normal to the wall.

## Simple Theory of Wall Pressure Fluctuations

We should begin by noting that the theory which follows is devoted to wall pressure effects and is not concerned with radiated noise, wake noise, or propulsor noise. Regardless of the source of noise, however, the fluctuating pressure in an incompressible turbulent flow may be calculated, at least in principle, from the Navier-Stokes equation:

$$\partial\bar{V}/\partial t + (\bar{V}\cdot\Delta)\bar{V} + (1/\rho)\Delta(P) = \text{viscous terms} \qquad (6)$$

where $\bar{V}$ and $P$ are instantaneous velocity and pressure and $\Delta$ is the vector gradient operator. The viscous terms for a Newtonian fluid are simply $\nu\Delta^2\bar{V}$. Polymer solutions, however, are non-Newtonian (drag reduction being one obvious departure from the Newtonian) but, if sufficiently dilute, may show a linear stress/strain-rate relation in simple shear.

A scalar differential equation for the fluctuating pressure component $p$ is obtained by taking the divergence of Eq. (6) with the result[5]

$$\Delta^2 p = -2\rho\frac{\partial U_i}{\partial x_j}\frac{\partial u_j}{\partial x_i} - \rho\frac{\partial^2}{\partial x_i \partial x_j}(u_iu_j - \overline{u_iu_j}) + \Delta\cdot\text{viscous terms} \qquad (7)$$

where the overbar denotes the time mean and where $U_i$ and $u_i$ are the mean and fluctuating velocities, respectively. In taking the divergence of Eq. (6), the unsteady acceleration and the linear viscous terms will vanish because of the incompressible continuity requirement. Only nonlinear viscous terms would remain, but for polymer solutions these would be necessarily very small at the extremely low concentrations considered. Thus, we conclude that the Laplacian of the pressure is essentially due only to the divergence of the instantaneous convective acceleration. Equation (7) is Poisson's equation, where the right-hand side may be thought of as the "pressure sources," one of which is linear and the other quadratic in the velocity of fluctuations. No quantitative calculations can be made unless the statistical properties of these sources are known, although Kraichnan,[6] Hodgson,[7] Gardner,[8] and White[9] gained some information by roughly averaging the source function. The quadratic terms are beyond measurement by present techniques, but, fortunately, it is generally believed that the first (or linear) term in Eq. (7) is the dominant one. Willmarth[21] and Hodgson[22] give plausible arguments to support this belief, but Corcos,[5] finds that the quadratic terms are also important. We assume here—justified to some extent by the final results—that the quadratic terms are negligible to reasonable approximation.

We now confine our attention to the turbulent boundary layer with zero mean pressure gradient. It is convenient to adopt a Cartesian system $(x,y,z)$ such that $x$ is parallel to a uniform freestream and $y$ is normal to the wall, so that $U_i = [U(y),0,0]$ and $u_i = (u,v,w)$. In this system, the linear terms in Eq. (7) have but a single nonvanishing component, so that the fluctuating pressure now satisfies the approximate relation

$$\Delta^2 p \doteq -2\rho(dU/dy)(\partial v/\partial x) \qquad (8)$$

According to this theory, wall pressure fluctuations in the boundary layer are primarily caused by longitudinal changes in the normal velocity fluctuation $v$, for which sufficient data exist to enable the solution of Eq. (8) to be given in statistical terms. As is well known, Poisson's equation has a formal integral solution, and Kraichnan[6] has shown that the normal derivative $(\partial p/\partial y)$ is negligibly small at the wall. Therefore, Eq. (8) has the following solution for pressure $p_w$ at any point $\mathbf{X}$ on the wall:

$$p_w(\mathbf{X}, t) = \frac{\rho}{\pi}\int_{y\geq 0}\frac{(dU/dy)(\partial v/\partial x)\,dW}{|\mathbf{X}-\mathbf{W}|} \qquad (9)$$

where the integration is carried out in the infinite half-space above the wall. Since $p_w$ is a random function, we are concerned with its space/time correlation function $R_p$, defined as

$$R_p(\xi,\eta,T) = \overline{p_w(x,0,z,t)\,p_w(x+\xi,0,z+\eta,t+T)} \qquad (10)$$

where $\xi$, $\eta$, and $T$ are the longitudinal, lateral, and time separations of any two correlated points on the wall. The total magnitude of the flow noise is the mean square wall pressure $p_w^2 = R_p(0,0,0)$. The calculation of $R_p$ from Eq. (9) will obviously result in a double integral involving the correlation function $\overline{(\partial v/\partial x)(\partial v'/\partial x')}$ which therefore must be a known function of separation between any two correlated points above the wall. Fortunately, Gardner[8] discovered a clever way to eliminate the horizontal variations of this function from the integrand. This procedure, which uses Hankel transforms, is outlined in detail in Refs. 8 and 9 and only the final result is repeated here:

$$R_p = 16\rho^2\int_0^\infty y\left(\frac{dU}{dy}\right)^2 dy\int_0^\infty\frac{dk}{k}e^{-2ky}f(k,y)J_0(Q) \qquad (11)$$

where $Q = k[(\xi - UT)^2 + \eta^2]^{1/2}$ and $J_0$ is the Bessel function of the first kind. The problem now reduces to the determination of the function $f(k,y)$, which is the Hankel transform over horizontal separation of the space correlation of $(\partial v/\partial x)$:

$$f(k,y) = \int_0^\infty h\, dh\, J_0(kh)\, \overline{\frac{\partial v}{\partial x}\frac{\partial v'}{\partial x'}} \tag{12}$$

Unfortunately, the correlation of $(\partial v/\partial x)$ has not been measured for horizontal separations, even for a Newtonian fluid. However, its mean value (zero separation) has been measured by Klebanoff,[23] and separation information can be calculated from the normal velocity spectrum $f_v(k_v)$, which is available from several studies.[21,23–25] Since the horizontal correlation scale of $(\partial v/\partial x)$ is much smaller than that of $v$ itself, we may use the Taylor hypothesis $\partial/\partial x \doteq (1/U)(\partial/\partial t)$ to calculate the function we need in Eq. (12) from $f_v(k_v)$:

$$R_v(h) = \overline{\frac{\partial v}{\partial x}\frac{\partial v'}{\partial x'}} \doteq \int_0^\infty k_v^2 f_v(k_v) \cos(k_v h)\, dk_v \tag{13}$$

The only restriction is that $R_v$ must decay much faster than $f_v$, which is the case. Figure 2 shows the measured $v$ spectra of Klebanoff[21] at two $y$ positions. The data are not normalized, i.e., the integral under the curves equals $\overline{v^2}$. Note the loss of high-frequency content as one moves away from the wall, which will cause the scale of $R_v(h)$ in Eq. (13) to increase with $y$. In principle, then, the problem of calculating the statistical properties of wall flow noise is resolved: we may use $f_v$ data to obtain $R_v$ from Eq. (13), use $R_v$ to get $f(k,y)$ from Eq. (12), and then use $f(k,y)$ in Eq. (11) to obtain the pressure correlation $R_p(\xi, \eta, T)$. The mean velocity derivative $(dU/dy)$ follows from Eq. (1), which, however, is invalid in the viscous sublayer. A better formula, accurate right down to the wall, obtains from Spalding's[26] inverse relationship

$$y^+ = u^+ + e^{-kB}\left(e^s - 1 - s - \frac{s^2}{2} - \frac{s^3}{6} - \frac{s^4}{24}\right)$$
$$s = ku^+ \tag{14}$$

Differentiating and neglecting a fourth-order term, we have

$$dU/dy = v^*/[ky + (\nu/v^*)(1 - ku^+)] \tag{15}$$

The term involving $\nu$ is the contribution of the sublayer, which we shall see amounts to only a small fraction of the total flow noise.

The calculation of $R_p$ involves 4 successive numerical quadratures. To evaluate the mean square noise $R_p(0,0,0)$, it suffices to approximate the integrands by simple functions. The curves in Fig. 2 can be fit by the expression

$$f_v(k_v) \doteq C/(k_v^2 + a)(k_v^2 + b) \tag{16}$$

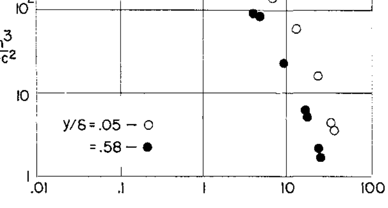


**Fig. 2 Flat-plate normal velocity spectra as measured by Klebanoff.**[21]

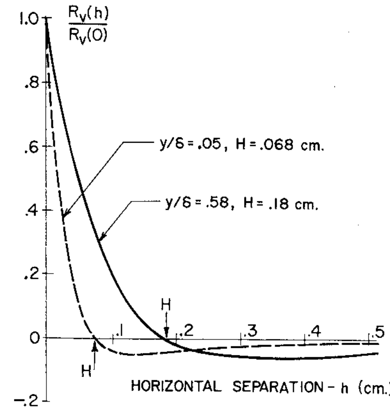


**Fig. 3 The normalized horizontal correlation $R_v(h)$ from Eq. (17) for the spectra of Fig. 2.**

where $a$, $b$, and $C$ vary with $y$. Equation (13) is then readily integrated:

$$R_v(h) = [\pi C/2(b - a)](b^{1/2}e^{-hb^{1/2}} - a^{1/2}e^{-ha^{1/2}}) \tag{17}$$

Figure 3 illustrates the two curves for $R_v(h)$ which are obtained from Eq. (17) after fitting the data of Fig. 2 to values of $a$, $b$, and $C$. Note the rapid decay of $R_v$ in Fig. 3 as compared to Klebanoff's 8-cm boundary-layer thickness. All such $R_v$ curves calculated by the author resulted in such similar shapes that they can all be characterized by a single "scale" parameter, their zero-crossing point $H(y)$, illustrated in Fig. 3. The final flow noise formula is thus written in terms of $H(y)$.

With $R_v$ given by exponentials in $h$, Eq. (12) is easily evaluated from the known Hankel transform (Ref. 27, p. 28):

$$\int_0^\infty h\, dh\, J_0(kh)e^{-h/L} = L^2(1 + k^2L^2)^{-3/2} \tag{18}$$

To avoid the fractional powers, we choose to expand $f(k,y)$ as a polynomial in $(k^2)$, the first coefficient of which correlates gratifyingly with the zero-crossing parameter $H(y)$:

$$f(k,y) \doteq 3.8H^4R_v(0) + \mathcal{O}(k^4)\ldots \tag{19}$$

The terms of order $(k^4)$ have a negligible effect on $R_p(0,0,0)$ but must be retained if $\xi$, $\eta$, or $T$ are not zero. For zero separation, $J_0(Q) = 1.0$ in Eq. (11), and the first term of Eq. (19) yields a simple result for the inner integral of $R_p$:

$$\int_0^\infty \frac{dk}{k} e^{-2ky} f(k,y) \doteq \frac{3.8H^4R_v(0)}{4y^2} \tag{20}$$

Finally, the outer integral of Eq. (11) can be nondimensionalized with respect to the shear velocity $v^*$ and the boundary-layer thickness $\delta$. Using the notation $y^* = y/\delta$, we have the final desired formula for calculating the mean wall pressure beneath a turbulent boundary layer (assuming that $k = 0.4$):

$$R_p(0,0,0) = \overline{p_w^2} = 95\tau_w^2 \int_0^\infty \frac{R^*(H/\delta)^4\, dy^*}{y^*[y^* + (2.5 - u^+)/\delta^+]^2} \tag{21}$$

where $R^* = \delta^2R_v(0)/v^{*2}$ is the dimensionless mean square of the fluctuating quantity $(\partial v/\partial x)$. The quantity $\delta^+ = \delta v^*/\nu$ is a measure of the viscous sublayer contribution to wall noise. Equation (21), although admittedly the result of many successive approximations, is an important result, because it gives insight into the mechanism of wall noise production. It appears that there are 4 separate parameters, listed here in order of decreasing importance: 1) the velocity derivative scale, $H$; 2) the wall shear stress, $\tau_w$; 3) the velocity derivative magnitude, $R_v(0)$; 4) the dimensionless thickness $\delta^+$ (Reynolds number). The parameter $H$ is enhanced by its occurrence to the 4th power. The thickness $\delta^+$ is of little importance.

We shall now apply Eq. (21) to three examples: zero pressure gradient, adverse pressure gradient, and an aqueous

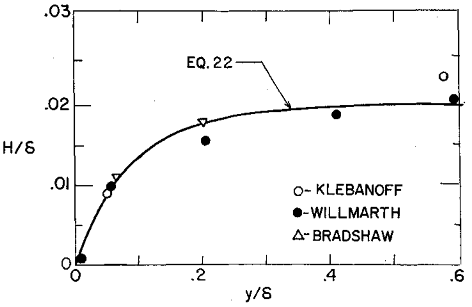


**Fig. 4 Calculated values of the velocity derivative scale $H(y)$ for flat-plate flow.**

solution of polyox WSR-301. The relative effect of $H$, $\tau_w$, and $R_v(0)$ shifts greatly among these three examples.

## Application to Newtonian Flat-Plate Flow

Most of the wall noise measurements in the literature are for zero pressure gradient, and it is generally agreed in this case that the rms wall pressure is approximately twice the wall shear stress: $p_w \doteq K\tau_w$, $K = 2.0 \rightarrow 2.5$. For our analysis, we can supplement the two curves of Fig. 2 with 7 other $v$ spectra from Willmarth[21] and Bradshaw.[25] This results in 9 values of $H(y)$ from Eq. (17) as plotted in Fig. 4. The trend is consistent in spite of the 3 different sources of data, and we may fit a curve to the points

$$H/\delta \doteq 0.02(1 - e^{-12y^*}) \qquad (22)$$

We may also compute 9 values of $R_v(0)$ from Eq. (17), and these scatter reasonably about the measured values of Klebanoff,[23] which can be fitted by an exponential:

$$R^* = \delta^2 R_v(0)/v^{*2} \doteq 5600.\ e^{-4y^*} \qquad (23)$$

Equations (22) and (23) may be substituted into Eq. (21) and the integration performed for various values of the thickness $\delta^+$. Some numerical results are given in Table 1. The computed values of $(p_w/\tau_w)$ are very similar in shape to the measurements reported by Corcos[5] but are about 20% lower. Presumably the difference is made up by the quadratic velocity terms which were neglected in Eq. (7). The general space/time correlations $R_p(\xi, \eta, T)$ as computed from Eqs. (11, 22, and 23) are also in agreement with measured values but will not be shown here. We conclude from this case that the present "linear source" theory is sufficiently justified to attempt other examples.

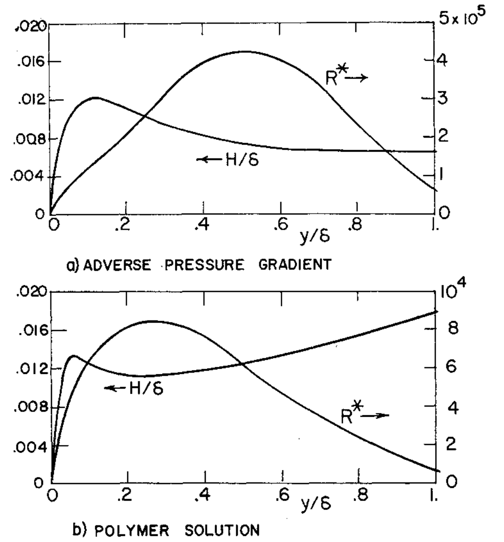


**Fig. 5 Calculated values of $H$ and $R_v(0)$ for a) the adverse pressure gradient of Bradshaw[25] and b) the polyox solution of Barchi[19] (Fig. 6).**

**Table 1 Flat-plate mean wall pressure from Eq. (21)**

| $\delta^+$ | Equivalent $R_x$ | $p_w/\tau_w$ |
|---|---|---|
| 1000 | $8.1 \times 10^5$ | 2.34 |
| 3000 | $3.2 \times 10^6$ | 1.93 |
| 6000 | $7.6 \times 10^6$ | 1.78 |
| 10000 | $1.4 \times 10^7$ | 1.72 |
| $\infty$ | $\infty$ | 1.69 |

## Application to an Equilibrium Adverse Gradient

Measurements of wall noise in an adverse pressure gradient have been reported by Bradshaw[25] and by Schloemer.[28] The adverse gradient reduces $\tau_w$ but causes higher turbulence intensities which are distributed across the boundary layer instead of being concentrated at the wall as in flat-plate flow. For example, Klebanoff[23] reports a maximum normal velocity fluctuation $(v/U) = 4\%$ at $y^* = 0.2$, whereas Bradshaw's decelerating flow shows a maximum of 7% at $y^* = 0.5$. Also, the $v$ spectra of Bradshaw[25] are twice as high as those of Klebanoff in Fig. 2 and do not lose high-frequency content at large $y^*$. Unfortunately, Schloemer[28] gave no $v$ spectra, so that Eq. (21) cannot be compared with his measured $p_w$. The strength of the adverse gradient can be characterized by Clauser's[29] parameter $\beta = \delta^*(dp/dx)/\tau_w$, which equals zero for a flat plate, 3.3 for Schloemer,[28] and 5.3 for Bradshaw.[25] Bradshaw's flow was a true "equilibrium" boundary layer (constant $\beta$), but Schloemer's was not. With respect to our noise relation, Eq. (21), we may generalize that an adverse pressure gradient makes $H(y)$ more uniform, decreases $\tau_w$, and increases $R_v(0)$ and $\delta^+$. Values of $H$ and $R_v(0)$ as calculated from Bradshaw's $v$ spectra are shown in Fig. 5. Note that $R^*$ is 20 times higher than for flat-plate flow. Figure 5 can be combined with Eq. (21) and Bradshaw's $U(y)$, which is still approximately logarithmic, and graphical integration performed to yield $(p_w/\tau_w) \doteq 8.8$, whereas Bradshaw reported a measured value of 8.0. Some tabulated theory and data for adverse pressure gradient are given in Table 2. Table 2 also relates $p_w$ to the freestream dynamic pressure $q = \frac{1}{2}\rho U^2$ through the skin-friction definition $C_f = \tau_w/q$. No single value could be given for a flat plate because of the variation of $C_f$ with $R_x$, but a rough estimate would be $p_w/q \doteq 0.006$, corresponding to $p_w = 2\tau_w$ and $C_f = 0.003$. Bradshaw's data appear to give further justification to the present analysis.

## Application to a Polymer Solution

Although polymer skin-friction reduction is well documented, the author knows of no measurements in the literature of either wall pressure or $v$ spectra. However, $u$ spectra are given for a guar gum solution of CMC (sodium carboxymethylcellulose) by Wells[30] and for polyox WSR-301 by Johnson and Barchi,[31] with further polyox data in the thesis of Barchi.[32] When compared to Newtonian spectra, both CMC and polyox show the same effect: a loss of high-frequency content near the wall and a gain away from the wall. This is illustrated by Barchi's data in Fig. 6. Since Klebanoff[23] has shown for Newtonian flow that both $u$ and $v$ spectra behave in nearly the same manner across the boundary layer, we may, without belaboring the point, use the data in Fig. 6 in Eq. (21) to make a crude estimate of wall noise in a polymer

**Table 2 Mean wall pressure for adverse pressure gradient**

| $\beta = \frac{\delta^*}{\tau_w}\frac{dp}{dx}$ | Ref. | $p_w/\tau_w$ | | $p_w/q$ | |
|---|---|---|---|---|---|
| | | Data | Eq. (21) | Data | Eq. (21) |
| 0.0 | 5 | 2.0–2.5 | 1.7–2.3 | ... | ... |
| 3.3 | 28 | 4.3 | ... | 0.0078 | ... |
| 5.3 | 25 | 8.0 | 8.8 | 0.010 | 0.011 |

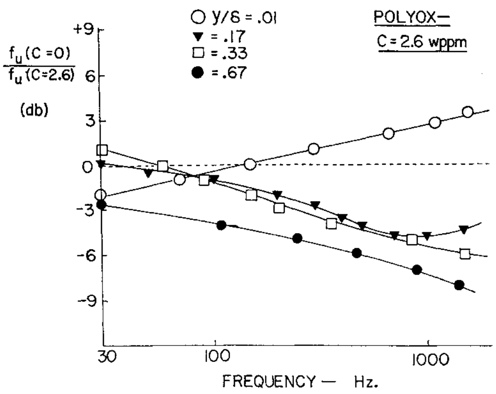


**Fig. 6 Longitudinal velocity spectral density ratio with and without polyox injection, after Barchi.[19]**

solution. The result may serve as a guide until polymer noise data are available.

Values of $H(y)$ and $R^*(y)$, calculated from the data of Fig. 6, are shown in Fig. 5b. The polyox seems to increase both $H$ and $R^*$ near the wall, and evaluation of the integral in Eq. (21) gives $p_w \doteq 4.1\tau_w$, or about 50% more noise than plain water. We may also relate $p_w$ to $q$ through Eq. (4). Barchi's experimental Reynolds number was $R_x = 2.6 \times 10^6$, and, for a polyox concentration $C \doteq 2.6$ wppm, Fig. 1 predicts $\alpha \doteq 3.7$. With $R_x$ and $\alpha$ known, Eq. (4) predicts that $Z = 29.3$, or $C_f = 2/Z^2 = 0.0023$—a 25% friction reduction over plain water. Thus, for this polyox experiment, the present analysis predicts $p_w/q = 4.1(0.0023) = 0.0095$, compared to the plain-water value of 0.0075 in this case. A similar result obtains from the CMC data of Wells,[30] but the noise increase is smaller. Although this is admittedly sketchy data, we can surmise that polymer additives, which definitely cause friction reduction, also cause such changes in the normal velocity spectra as to probably rule against noise reduction. It is clear that more data are needed.

## Conclusions

By making suitable assumptions about the source of pressure fluctuations and their distribution with respect to normal velocity spectra, a simple theory has been derived [Eqs. (11) and (21)] for the statistical properties of wall flow noise. Comparison with Newtonian data for flat plates, pipes, and adverse pressure gradients show good agreement and lend support to the simple theory. Based on sketchy velocity data for polyox and CMC solutions, a tentative calculation is made which predicts that such additives do not cause noise reduction.

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